

Hard four jet production in pA collisions

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In a suitably chosen back-to-back kinematics, four-jet production in hadronic collisions is known to be dominated by contributions from two independent partonic scattering processes, thus giving experimental access to the structure of generalized two-parton distributions ($_2$ GPDs). Here, we show that a combined measurement of the double hard four-jet cross section in proton-proton and proton-nucleus collisions will allow one to disentangle different sources of two-parton correlations in the proton, that cannot be disentangled with 4-jet measurements in proton-proton collisions alone. To this end, we analyze in detail the structure of $_2$ GPDs in the nucleus (A), we calculate in the independent nucleon approximation all contributions to the double hard four-jet cross section in pA, and we determine corrections arising from the nuclear dependence of single parton distribution functions. We then outline an experimental strategy for determining the longitudinal two-parton correlations in the proton.

PACS numbers:

Keywords:

I. INTRODUCTION

With increasing center of mass energy at hadron colliders, multi-particle final states at high transverse momentum receive an increasingly important contribution from multi-parton interactions. The prototype of such processes are double parton interactions in which two partons from each hadron enter into collision at two distinct hard vertexes. The calculation of such double parton interactions involves double Generalized Parton Distributions ($_2$ GPDs) that - in principle - contain information about the spatial and momentum correlations of two partons in the incoming hadronic wave function. In practice, experimental constraints on these correlations are scarce. Since nuclear projectiles offer significantly different two-parton correlations, the question arises to what extent a combined analysis program of proton-proton and proton-nucleus collisions at the LHC could help to constrain $_2$ GPDs. In this paper, we classify the different contributions to nuclear $_2$ GPDs and how they contribute to the double parton scattering cross section into 4 jets that is experimentally accessible in the upcoming p-A run at the LHC.

We focus on the physics in a suitably chosen back-to-back kinematics: the dijet momentum imbalances $\delta_{13}^2, \delta_{24}^2$ are $\delta_{13}^2 = (\vec{k}_{1t} + \vec{k}_{3t})^2 \ll k_{1t}^2 \sim k_{3t}^2 \sim Q_1^2, \delta_{24}^2 = (\vec{k}_{2t} + \vec{k}_{4t})^2 \ll k_{2t}^2 \sim k_{4t}^2 \sim Q_2^2$, where the momenta $\vec{k}_{1t}, \vec{k}_{2t}, \vec{k}_{3t}, \vec{k}_{4t}$ are the transverse momenta of individual jets, the scales Q_1, Q_2 are the resolution scales of two partons. In this kinematics the production of four jets in the collision of two partons (so-called $2 \rightarrow 4$ process) is suppressed in the leading logarithmic approximation compared to the hard processes that involve the collision of four partons [1–3]. For this back-to-back kinematics, it is useful to write $_2$ GPDs as the sum of two contributions,

$$_2G_p(x_1, x_2, Q_1^2, Q_2^2, \vec{\Delta}) = G_p^{\text{double}}(x_1, x_2, Q_1^2, Q_2^2, \vec{\Delta}) + G_p^{\text{single}}(x_1, x_2, Q_1^2, Q_2^2, \vec{\Delta}). \quad (1)$$

Here, $_2$ GPDs are written as functions of the momentum fractions x_1, x_2 and the resolution scales Q_1, Q_2 of the two partons. In the following, we will not write explicitly the dependence of $_2$ GPDs and GPDs on resolution scales. The transverse momentum parameter $\vec{\Delta}$ denotes the difference between the transverse momenta exchanged by one parton in the amplitude and complex conjugate amplitude and it is conjugate to the relative spatial transverse distance between the two partons. In general, $_2$ GPDs contain a non-perturbative two-parton contribution in which QCD evolution amounts to an independent evolution of both partons with standard one-parton evolution equations (G_p^{double}). In addition, there is a contribution in which both partons result perturbatively as the two daughters of a single parent parton in the QCD evolution (G_p^{single}). This second term couples the evolution of $_2$ GPDs to the evolution

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of the standard one-parton distribution functions. For the production of two pairs of jets in independent back-to-back kinematics, both contributions are known to contribute with parametrically equal weights [3].

The double hard four jet cross section for the collision of hadrons A and B can then be written in terms of $_2\text{GPDs}$ as

$$\frac{d\sigma_{4jet}^{AB}}{d\hat{t}_1 d\hat{t}_2} = \int \frac{d^2\vec{\Delta}}{(2\pi)^2} \frac{d\hat{\sigma}_1(x'_1, x_1)}{d\hat{t}_1} \frac{d\hat{\sigma}_2(x'_2, x_2)}{d\hat{t}_2} {}_2G_A(x'_1, x'_2, \vec{\Delta}) {}_2G_B(x_1, x_2, \vec{\Delta}). \quad (2)$$

We denote by $\hat{\sigma}_i$ the partonic $2 \rightarrow 2$ scattering cross sections. The measurable four-jet cross section in AB collisions, σ_{4jet}^{AB} , is a function of the four jets' c.m. transverse energy and rapidity, that are connected to the variables x_i, x'_i and the virtualities in the standard way. For the case that $\hat{\sigma}_1$ and $\hat{\sigma}_2$ denote indistinguishable scattering processes, equation (2) must be multiplied by a symmetry factor $1/2$ that we omit for brevity. In proton-proton collisions, it is customary to parametrize the cross section σ_{4jet}^{pp} as the product of two two-jet cross sections σ_{2jet}^{pp} ,

$$\frac{d\sigma_{4jet}^{pp}}{d\hat{t}_1 d\hat{t}_2} = \frac{1}{S} \frac{d\sigma_{2jet}^{pp}}{d\hat{t}_1} \frac{d\sigma_{2jet}^{pp}}{d\hat{t}_2}. \quad (3)$$

Here the quantity S (sometimes referred to as σ_{eff}) characterizes the effective transverse area of the four parton interaction and the effect of longitudinal correlations between the partons in the colliding hadrons. In general, S can depend on the momentum fractions x_i and virtualities of the incoming partons. Data from the Tevatron indicate that S is typically of the order of 15 mb [4–7]. This is a factor ~ 2 smaller than expectations based on uncorrelated two-parton distributions, see e.g. [8]. It is a clear indication that non-trivial parton correlations are experimentally accessible in double hard parton interactions. To date, it remains an open question of whether these two-parton correlations in the proton are predominantly transverse (as realized e.g. in models that picture the proton as composed of several hot spots) or predominantly longitudinal (as resulting e.g. from perturbative $1 \rightarrow 2$ splittings). These and other questions about the Tevatron data have motivated much work recently [9–25]. Older relevant work includes [26, 27].

The case of double hard four jet production in proton-nucleus collisions was discussed first in [28] where it was pointed out that such measurements would be sensitive to the longitudinal correlations of the partons. More recently, multiple parton interactions were considered in the production of two leading pions in deuteron-gold collisions at RHIC [29], and for the case of proton-deuteron collisions [30]. In the present paper, we aim to extend this discussion in the light of the recent pQCD studies [1–3] mentioned above. For the case that A is a nucleus, the $_2\text{GPD}$ may be written as the sum of three distinct contributions, depending on whether both partons belong to the same nucleon (1N) or to different nucleons (2N). Distinguishing for the first case again single from double contributions, as in eq. (1), one obtains

$${}_2G_A(x_1, x_2, \vec{\Delta}) = G_A^{\text{single}, 1N}(x_1, x_2, \vec{\Delta}) + G_A^{\text{double}, 1N}(x_1, x_2, \vec{\Delta}) + G_A^{2N}(x_1, x_2, \vec{\Delta}). \quad (4)$$

As will become clear in the following, this classification relies on viewing the nucleus as a superposition of nucleons to which partons can be assigned uniquely. Inserting the expressions for $_2\text{GPDs}$ inside a nucleus (4) and inside a nucleon (1) into the double hard four jet cross section (2), the following different contribution emerge for the case of proton-nucleus scattering:

- I. $G_p^{\text{double}} \otimes G_A^{\text{double}, 1N}$
- II. $G_p^{\text{double}} \otimes G_A^{\text{single}, 1N}$
- III. $G_p^{\text{double}} \otimes G_A^{2N} \Big|_{\text{direct}}$
- IV. $G_p^{\text{single}} \otimes G_A^{\text{single}, 1N}$
- V. $G_p^{\text{single}} \otimes G_A^{\text{double}, 1N}$
- VI. $G_p^{\text{single}} \otimes G_A^{2N} \Big|_{\text{direct}}$

The terms in which two partons are taken from the same nucleon in a nucleus contribute equally to proton-nucleon collisions. More specifically, these are the $4 \rightarrow 4$ contribution (case I) and the $3 \rightarrow 4$ contributions (case II and V). They will be discussed in section II. It is known that the single-single contribution (case IV) does not make a dominant contribution to the back-to-back kinematics; rather, it can be regarded as a one-loop correction to the $2 \rightarrow 4$ cross section [1].

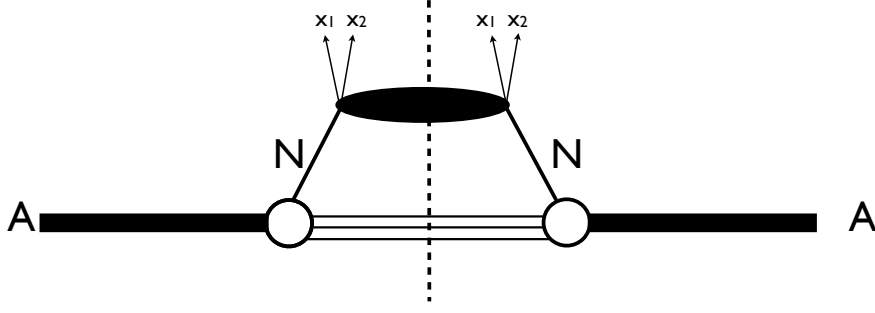


FIG. 1: Schematic representation of the contributions $G_A^{\text{single}, 1N} + G_A^{\text{double}, 1N}$ to the nucleus $_2\text{GPD}$ that enter the cross section for double hard 4-jet production (cases I, II and IV, V discussed in the text). Two partons of momentum fractions x_1, x_2 are drawn from the same single nucleon in the amplitude (left hand side of the diagram) and complex conjugate amplitude. Depending on whether the two nucleons arise perturbatively in a $1 \rightarrow 2$ splitting from a single parton in this nucleon, or whether they are of non-perturbative origin, we shall refer to them as $G_A^{\text{single}, 1N}$ and $G_A^{\text{double}, 1N}$, respectively.

For a nuclear projectile, two additional contributions arise. These involve two partons from different nucleons in the nucleus that interact with G_p^{double} (case III) or G_p^{single} (case VI) of the proton, respectively. These will be discussed in section III. We have labeled both these contributions with the subscript 'direct' to indicate that the partons with momenta x_1 and x_2 are assigned to the *same* nucleons in amplitude and complex conjugate amplitude.

Viewing the nucleus as a bound state of many nucleons (without considering modifications to their internal structure) is an approximation. If we relax the working hypothesis that partons can be assigned uniquely to nucleons in a nucleus, then additional contributions are possible. First, it is possible that two nucleons of the nuclear wave function are involved in both amplitude and complex conjugate amplitude, but that the two partons are interchanged across the cut. We label these contributions with the subscript 'interference'. Second, it is conceivable that the two partons taken from the nucleus belong to one nucleon in the amplitude but to two different nucleons in the complex conjugate amplitude.

VII. $G_p^{\text{double}} \otimes G_A^{2N} \Big|_{\text{interference}}$

VIII. $G_p^{\text{single}} \otimes G_A^{2N} \Big|_{\text{interference}}$

IX. 1N-2N interference

In general, such interference terms indicate the break-down of a probabilistic picture of double-parton interactions in pA collisions in terms of an independent superposition of nucleons. By their very nature, they characterize partonic cross-talk between different nucleons in a nucleus and may thus help to elucidate partonic nuclear structure. We note that deviations from the simple picture of a nucleus as a superposition of nucleons are known already on the level of single parton distributions, e.g. as EMC and nuclear shadowing effects [31]. In the following sections, we discuss the different contributions to the double-hard four jet cross section following the classification listed above.

II. SINGLE NUCLEON SCATTERING TERMS (CASES I, II AND V)

For the contributions I-VI., the momenta of the nucleons are the same in amplitude and complex conjugate amplitude. Therefore, if both partons belong to the same nucleon (cases I, II and V), one can integrate over the momenta of all other nucleons and write the corresponding part of the nuclear $_2\text{GPD}$ as

$$G_A^{1N}(x_1, x_2, \vec{\Delta}) = \int \frac{1}{\alpha^2} \left(G_N^{\text{single}}\left(\frac{x_1}{\alpha}, \frac{x_2}{\alpha}, \vec{\Delta}\right) + G_N^{\text{double}}\left(\frac{x_1}{\alpha}, \frac{x_2}{\alpha}, \vec{\Delta}\right) \right) \rho_A^N(\alpha, p_t) \frac{d\alpha}{\alpha} d^2 p_t. \quad (5)$$

Here, the nuclear $_2\text{GPD}$ is the sum of the terms $G_A^{\text{single}, 1N}$ and $G_A^{\text{double}, 1N}$, introduced before, and expressed as an integral over the corresponding contributions of individual nucleons. The quantity $\rho_A^N(\alpha, p_t)$ denotes the light-cone nucleon density of the nucleus normalized as $\int \rho_A^N(\alpha, p_t) d\alpha/\alpha = A$. The factor $1/\alpha$ for each of the partons reflects the fact that the number of partons between x_1 and x_2 should not change under Lorentz boosts. These extra factors $1/\alpha$ are absorbed in the flux such that one recovers for the structure function the standard expression $F_{2A}(x, Q^2) = \int \rho_A^N(\alpha, p_t) \frac{d\alpha}{\alpha} d^2 p_t F_{2N}(\frac{x}{\alpha}, Q^2)$.

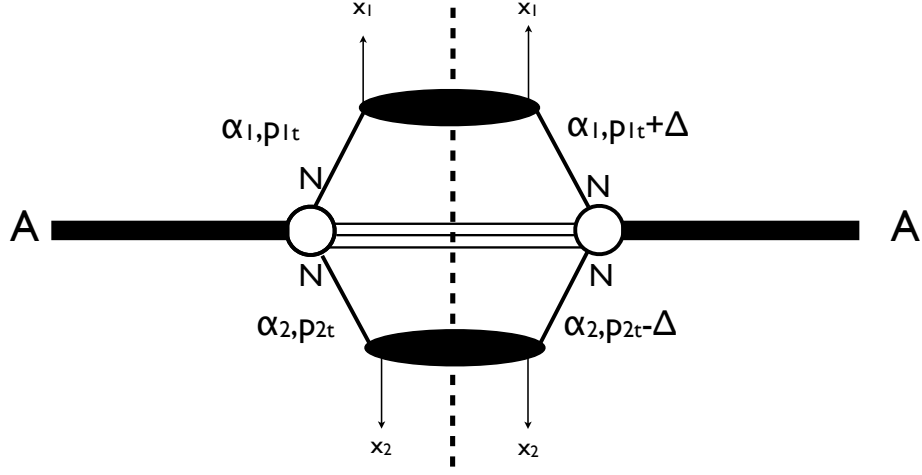


FIG. 2: Schematic representation of the contribution G_A^{2N} to the nucleus 2 GPD that enters the cross section for the double hard 4-jet production in the terms III and VI discussed in the text. Here, the two partons are drawn from two different nucleons that carry longitudinal nucleon momentum fractions α_1 and α_2 .

Equation (5) is written for an ensemble of A moving nucleons satisfying the momentum sum rule $\int \alpha \rho_A^N(\alpha, p_t) d\alpha / \alpha d^2 p_t = A$ since $\sum_i \alpha_i = A$. This raises the question of how well one can approximate G_A^{1N} in terms of the distributions $G_N^{single}(x_1, x_2, \vec{\Delta}) + G_N^{double}(x_1, x_2, \vec{\Delta})$ written in the nucleus rest frame. To address this question, we replace $\alpha = 1 + (\alpha - 1)$ in the arguments of $(1/\alpha^2) G^{1N}(\frac{x_1}{\alpha}, \frac{x_2}{\alpha}, \vec{\Delta})$, and we expand in powers of $(\alpha - 1)$. Using momentum sum rule and baryon number sum rule, we find $\int \frac{d\alpha}{\alpha} (\alpha - 1) \rho_A^N(\alpha) = A - A = 0$, see Ref. [32, 33]. Therefore, corrections due to Fermi motion (i.e., corrections due to the α -dependence of the integrand of G_A^{1N}) arise only to second order in $(\alpha - 1)$. The longitudinal momentum distribution of nucleons in a nucleus peaks at $\alpha = 1$ with small dispersion, and therefore

$$G_A^{1N}(x_1, x_2, \vec{\Delta}) = A G_N(x_1, x_2, \vec{\Delta}) \left(1 + O \left(\int (\alpha - 1)^2 \rho_A^N(\alpha, p_t) \frac{d\alpha}{\alpha} d^2 p_t \right) \right). \quad (6)$$

Here, the correction term involves first and second derivatives of the nucleon 2 GPD with respect to x_1 and x_2 . A precise numerical estimate will have to constrain this term numerically. Parametrically, the correction is small. The dominant linear dependence of $G_A^{1N}(x_1, x_2, \vec{\Delta})$ on nucleon number A translates directly into a linear dependence of the corresponding contribution to the double hard four-jet cross section

$$\frac{\sigma_{4jet}^{pA, 1N}}{dt_1 dt_2} \approx A \frac{d\sigma_{4jet}^{pp}}{dt_1 dt_2} = \frac{A}{S} \frac{d\sigma_{2jet}^{pp}}{dt_1} \frac{d\sigma_{2jet}^{pp}}{dt_2}. \quad (7)$$

We have introduced an \approx sign in this relation to indicate that the identification of $\frac{d\sigma_{4jet}^{pA, 1N}}{dt_1 dt_2}$ with A times the corresponding cross section in pp relies on neglecting the nuclear modification of parton distribution functions. Our discussion up to section VI will rely on this approximation. This is justified since we consider larger Q^2 processes at moderate x , where nuclear modifications are expected to be small. Within this approximation, the contributions to the double hard four jet cross section in pA discussed here are exactly the contributions that one obtains from superimposing four jet cross sections from A independent nucleon-nucleon collisions; the effective transverse area S in (7) is therefore the quantity measured in pp collisions. In section VII, we go beyond this approximation and we discuss how the nuclear dependence of parton distribution functions can be taken into account.

III. DOUBLE NUCLEON SCATTERING TERMS (CASES III AND VI)

Figure 2 shows the 2 GPD contribution G_A^{2N} in which the two partons belong to two different nucleons in both amplitude and complex conjugate amplitude. This term enters the contributions III and VI of the double hard four

jet cross section. In terms of the nuclear light cone wave function ψ_A of the A -nucleon system, it takes the form

$$G_A^{2N}(x_1, x_2, \vec{\Delta}) = A(A-1) \int \frac{1}{\alpha_1 \alpha_2} \prod_{i=1}^{i=A} \frac{d\alpha_i d^2 p_{ti}}{\alpha_i} \delta\left(\sum_i \alpha_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\alpha_1, \alpha_2, p_{t1}, p_{t2}, \dots) \quad (8)$$

$$\times \psi_A(\alpha_1, \alpha_2, p_{t1} + \vec{\Delta}, p_{t2} - \vec{\Delta}, \dots) G_N(x_1/\alpha_1, |\vec{\Delta}|) G_N(x_2/\alpha_2, |\vec{\Delta}|).$$

Here, the transverse momentum transfer $\vec{\Delta}$ is exchanged between the two active nucleons, while their light cone fractions are conserved. The partonic momentum fractions x_i drawn from the two active nucleons are then determined via the generalized parton distributions $G_N(x_i/\alpha_i, \vec{\Delta})$ of the nucleons. Since the wave function ψ_A is normalized to unity, the prefactor $A(A-1)$ results from combinatorics. The factor $1/(\alpha_1 \alpha_2)$ has the same origin as the factor $1/\alpha^2$ in eq. (5).

Expanding in equation (9) the arguments of G_N in powers of $(\alpha_i - 1)$, the leading term can be written in a factorized form involving the two-nucleon form factor F_A^{double}

$$G_A^{2N}(x_1, x_2, \vec{\Delta}) = A(A-1) G_N(x_1, |\vec{\Delta}|) G_N(x_2, |\vec{\Delta}|) F_A^{\text{double}}(\vec{\Delta}, -\vec{\Delta}), \quad (9)$$

$$F_A^{\text{double}}(\vec{\Delta}, -\vec{\Delta}) = \int \prod_{i=1}^{i=A} \frac{d\alpha_i d^2 p_{ti}}{\alpha_i} \delta\left(\sum_i \alpha_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\alpha_1, \alpha_2, p_{t1}, p_{t2}, \dots) \quad (10)$$

$$\times \psi_A(\alpha_1, \alpha_2, p_{t1} + \vec{\Delta}, p_{t2} - \vec{\Delta}, \dots).$$

As for subleading correction, we note that one can exploit the symmetry of the integrand of (9) under $\alpha_i \rightarrow (2 - \alpha_i)$ in the nonrelativistic limit to see that corrections to (9) are of the order $(1 - \alpha)^2$, similar to the case of eq. (6). Since the momentum fraction α_i of all nucleons in the nucleus are close to unity, we can approximate them in the nucleus rest frame in the non-relativistic limit, $\alpha_i = 1 + p_{3i}/m_N$. The two-nucleon form factor reads then

$$F_A^{NR \text{ double}}(\vec{\Delta}, -\vec{\Delta}) = \int \left(\prod_{i=1}^{i=A} d^3 p_i \right) \psi_A^*(p_1, p_2, \dots) \psi_A(p_1 + \vec{\Delta}, p_2 - \vec{\Delta}, p_3, \dots) \delta^{(3)}\left(\sum_{i=1}^A p_i\right). \quad (11)$$

Equations (10) and (11) express the two-nucleon form factor for an arbitrary nucleus wave function, and therefore can account for arbitrary nucleon correlations. Also, when combined with eq.(9), these expressions can account for the finite size of nucleons as well (which is characterized by the single nucleon GPDs). We now discuss how more explicit expressions, suitable for direct numerical evaluation, can be obtained if assumptions about nucleon correlations and the finite size of nucleons are made.

First, we turn to the independent nucleon approximation, when the nuclear wave function is written as a product of single nucleon wave functions. This neglects all internucleon correlations, including constraints from recoil that arise from the kinematic δ -function in (11). (These latter corrections are proportional to $1/A$. A parametrically more important source of corrections to this picture of double hard 4-jet production arises from short-range NN interaction that are suppressed by a factor $\propto 1/A^{1/3}$.) One can express (11) in terms of products of Fourier transforms of single nucleon wave functions $\psi_N(r_i)$. Using a single nucleon density $\rho_A(r) = A \psi_N^*(r) \psi_N(r)$, that is normalized to $\int \rho_A(r) d^3 r = A$, the two-nucleon form factor is the product of single nucleon form factors

$$F_A^{\text{double}}(\vec{\Delta}, -\vec{\Delta}) \simeq \left| \int d^3 r \frac{1}{A} \rho_A(r) \exp[i\vec{\Delta} \cdot \vec{r}] \right|^2 = F_A(\vec{\Delta})^2. \quad (12)$$

Since $\vec{\Delta}$ is a two-dimensional vector in the transverse plane, the single nucleon form factor can be written in terms of the nuclear thickness function $T(b) = \int_{-\infty}^{\infty} dz \rho_A(z, \vec{b})$ as

$$F_A(\vec{\Delta}) = \frac{1}{A} \int d^2 b T(\vec{b}) \exp(i\vec{\Delta} \cdot \vec{b}). \quad (13)$$

The well-known approximate relation between the form factor and the nucleus radius R_A ,

$$F_A^{\text{double}}(\vec{\Delta}, -\vec{\Delta}) \approx \exp\left[-\frac{1}{3} \Delta^2 R_A^2\right] \quad (14)$$

can then be obtained by expanding (13) for small Δ , $F_A(\vec{\Delta}) \simeq 1 - \frac{1}{6} \Delta^2 R_A^2$ and reexponentiating this expression. However, the Gaussian approximation (14) somewhat underestimates the drop of $F_A(\Delta^2)$ with Δ^2 for $\Delta^2 R_A^2/6 \geq 1$.

So, while (14) is well-suited for parametric arguments, it is preferable to base numerical estimates on evaluating (13) without further approximation.

Second, we discuss now approximations that amount to neglecting the nucleon size in comparison to the nucleus size. According to (14), the double nucleon scattering contribution (9) to the $_2\text{GPD}$ has its main support for small values of $\Delta^2 < O(3/R_A^2)$. Parametrically, this is a factor $A^{-2/3}$ smaller than the range of Δ -values in which a nucleon $_2\text{GPD}$ has support. If one neglects the Δ -scale as being small, then the single GPDs become standard parton distributions and the $_2\text{GPDs}$ become two-parton distribution functions. In particular, the single nucleon contribution to the $_2\text{GPD}$ in (6) can be approximated as

$$G_A^{1N}(x_1, x_2, \vec{\Delta}) \simeq A G_N(x_1, x_2, \vec{\Delta}) \rightarrow A f_N(x_1, x_2), \quad (15)$$

where $f_N(x_1, x_2)$ is the standard two-parton distribution function. Similarly, the double nucleon contribution to the $_2\text{GPD}$ in (9) can be approximated as

$$G_A^{2N}(x_1, x_2, \vec{\Delta}) \rightarrow f_N(x_1) f_N(x_2) F_A^{\text{double}}(\vec{\Delta}, -\vec{\Delta}), \quad (16)$$

where $f_N(x)$ are standard single parton distribution functions. Simplified expressions for the double hard four jet cross section can then be obtained by inserting equations (15), (16) into (2).

- Case III

We consider first the $4 \rightarrow 4$ contribution to the double hard 4-jet cross section in which the double nucleon scattering term G_A^{2N} of the nuclear $_2\text{GPD}$ is paired with $G_p^{\text{double}}(x_1, x_2, \vec{\Delta})$ in the proton. We obtain

$$\frac{d\sigma_4^{(III)}(x'_1, x'_2, x_1, x_2)}{d\hat{t}_1 d\hat{t}_2} = A(A-1) \frac{d\hat{\sigma}_1}{d\hat{t}_1} \frac{d\hat{\sigma}_2}{d\hat{t}_2} \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} G_p^{\text{double}}(x'_1, x'_2, \vec{\Delta}) f_N(x_1) f_N(x_2) F_A^2(\vec{\Delta}). \quad (17)$$

Taking the nucleus large enough to ignore the nucleon size, see eq. (16), one can neglect the $\vec{\Delta}$ dependence of $G_p(x_1, x_2, \vec{\Delta})$. Then one can write

$$\frac{d\sigma_4^{(III)}(x'_1, x'_2, x_1, x_2)}{d\hat{t}_1 d\hat{t}_2} = A(A-1) \frac{d\sigma_1}{d\hat{t}_1} \frac{d\sigma_2}{d\hat{t}_2} \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} G_p^{\text{double}}(x'_1, x'_2, 0) f_N(x_1) f_N(x_2) F_A^2(\vec{\Delta}). \quad (18)$$

Here $G_p^{\text{double}}(x'_1, x'_2, 0) \equiv f_p(x'_1, x'_2)$ is the double parton distribution function, and f_N denotes standard nucleon pdfs. For a simple parametric estimate, the form factor $F_A^2(\vec{\Delta})$ can be viewed as a step function with support for $\Delta^2 < 3/R_A^2 \sim A^{-2/3}$. Therefore, the contribution (18) is $O(A^{4/3})$ which makes it $A^{1/3}$ -enhanced compared to all contributions discussed in section II. This can also be seen after Fourier transform to b -space, if one recalls that $T(b) \propto A^{1/3}$ for typical $b \ll R_A \sim A^{1/3}$,

$$\frac{\sigma_4^{(III)}(x'_1, x'_2, x_1, x_2)}{d\hat{t}_1 d\hat{t}_2} = \frac{f_p(x'_1, x'_2)}{f_p(x'_1) f_p(x'_2)} \frac{d\sigma_{2\text{jet}}^{pp}(x'_1, x_1)}{d\hat{t}_1} \frac{d\sigma_{2\text{jet}}^{pp}(x'_2, x_2)}{d\hat{t}_2} \frac{(A-1)}{A} \underbrace{\int T^2(b) d^2 b}_{\propto A^{4/3}}. \quad (19)$$

Here we expressed the four jet cross section in term of full dijet differential cross sections, defined through hard parton cross sections as:

$$\frac{d\sigma_{2\text{jet}}^{pp}}{d\hat{t}}(x'_1, x_1) = f_N(x_1) f_p(x'_1) \frac{d\hat{\sigma}}{d\hat{t}}(x'_1, x_1). \quad (20)$$

Except for the $(1 - 1/A)$ correction term, this form of the double hard 4-jet cross section was given first in [28]. We note that in evaluating $\int T^2(b) d^2 b$ in (19), short-range nucleon interactions can be taken into account. For $A \sim 200$, the resulting corrections are on the level of a few percent [35].

- Case VI

The double nucleon scattering term G_A^{2N} enters also in the $3 \rightarrow 4$ contribution to the double hard 4-jet cross section, where one parton of the proton splits into two partons with momentum fractions x'_1, x'_2 ,

$$\frac{d\sigma_4^{(VI)}}{d\hat{t}_1 d\hat{t}_2} = A(A-1) \frac{d\sigma_1}{d\hat{t}_1} \frac{d\sigma_2}{d\hat{t}_2} \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} G_p^{\text{single}}(x'_1, x'_2, 0) f_N(x_1) f_N(x_2) F_A^2(\vec{\Delta}) \propto A^{4/3}. \quad (21)$$

This term has the same parametric $A^{4/3}$ -enhancement as (18). However, as we explain now, its relative weight compared to $\sigma_4^{(III)}$ is significantly smaller in pA than the corresponding relative weight in pp collisions. To see this, let us recall first that in pp collisions the $4 \rightarrow 4$ contribution involves a Δ -integral over the fourth power of $F_N(\Delta)$. This follows for instance from writing in the mean field approximation each of the two $_2\text{GPDs}$ in (2) as the product of two single GPDs $_1G(x, Q^2, \vec{\Delta}) = f_N(x, Q^2) F_N(\Delta)$. On the other hand, in proton-proton collisions the Δ -integral of the $3 \rightarrow 4$ contribution involves only two powers of $F_N(\Delta)$, since $G_p^{single}(x_1, x_2, \vec{\Delta})$ corresponds to a point-like perturbative splitting and its Δ -dependence is thus negligible [1]. In general, since $F_N(\Delta)$ peaks at $\Delta = 0$ and falls off steeply with increasing Δ , the Δ -integral over $F_N^2(\Delta)$ is larger than that over $F_N^4(\Delta)$. This results in a geometrical enhancement of the $3 \rightarrow 4$ contribution in pp relative to the $4 \rightarrow 4$ contribution. For a numerical estimate of this enhancement in pp , one may take recourse e.g. to the $F(\Delta) = \frac{1}{(1+\Delta^2/m_g^2)^2}$ [36], for which

$$\int \frac{d^2\Delta}{(2\pi)^2} F_N^2(\Delta) / \int \frac{d^2\Delta}{(2\pi)^2} F_N^4(\Delta) = \frac{m_g^2}{12\pi^2} / \frac{m_g^2}{28\pi^2} = 7/3. \quad (22)$$

We note that this ratio is rather robust against changes of the functional shape of $F(\Delta)$; for instance, an exponential form of $F(\Delta)$ would yield a ratio 2 rather than 7/3. In contrast to this geometrical enhancement in pp , we have found here that in pA collisions in the limit of very large A , the Δ -integrals in (18) and (21) are the same and a geometrical enhancement as in (22) is missing. In addition, the single parton in a $3 \rightarrow 4$ process could belong to each of the colliding protons in pp , whereas this combinatorial factor 2 is obviously absent in pA . In summary, there is a geometrical enhancement factor of $3 \rightarrow 4$ relative to $4 \rightarrow 4$ processes in pp that equals $7/3 \times 2 = 14/3 \sim 5$ and that is clearly absent in pA collisions for sufficiently large A ,

$$\frac{\left. \frac{\sigma_4^{(VI)}}{\sigma_4^{(III)}} \right|_{pA}}{\left. \frac{\sigma_4^{(VI)}}{\sigma_4^{(III)}} \right|_{pp}} = \text{const}(A)_{|A \gg 1} \sim \frac{1}{5}. \quad (23)$$

IV. TOTAL CROSS SECTION IN INDEPENDENT NUCLEON APPROXIMATION.

In summary, working in the independent nucleon approximation and neglecting the nucleon size compared to the nuclear radius R_A , one can write the double hard four-jet cross section in pA as the sum of terms that are linear in A (see discussion of σ_4^{1N} in section II and eq. (7)) and of double scattering terms that were considered in section III (see eqs.(19), (21)). We define $\sigma_{4jet}^{(pA)}$ as the sum of all these contribution. Normalizing this sum by A times the corresponding cross section in a nucleon-nucleon collision, we obtain the 4-jet nuclear modification factor

$$\begin{aligned} R_{pA}^{4jet}(x_1, x_2, x'_1, x'_2) &\equiv \frac{d\sigma_{4jet}^{pA}(x_1, x_2, x'_1, x'_2)}{d\hat{t}_1 d\hat{t}_2} \bigg/ \frac{A}{S} \frac{d\sigma_{2jet}(x'_1, x_1)}{d\hat{t}_1} \frac{d\sigma_{2jet}(x'_2, x_2)}{d\hat{t}_2} \\ &= 1 + \frac{S}{A} \frac{A-1}{A} \int T^2(b) d^2b \frac{G_p(x'_1, x'_2)}{f_p(x'_1) f_p(x'_2)}, \end{aligned} \quad (24)$$

where

$$G_p(x'_1, x'_2) = f_p(x'_1, x'_2) + G_p^{single}(x'_1, x'_2, 0). \quad (25)$$

We note that a closely related expression was obtained already in the analysis of Ref. [28], where the $3 \rightarrow 4$ contribution was not included and $1/A$ -corrections were neglected. It is useful to write (24) in the compact form

$$R_{pA}^{4jet}(x_1, x_2, x'_1, x'_2) = 1 + S W(A) K(x'_1, x'_2), \quad (26)$$

where the second term on the right hand side of (24) factorizes into a product of the effective transverse area S , a purely geometrical overlap factor

$$W(A) = \frac{A-1}{A^2} \int d^2b T^2(b), \quad (27)$$

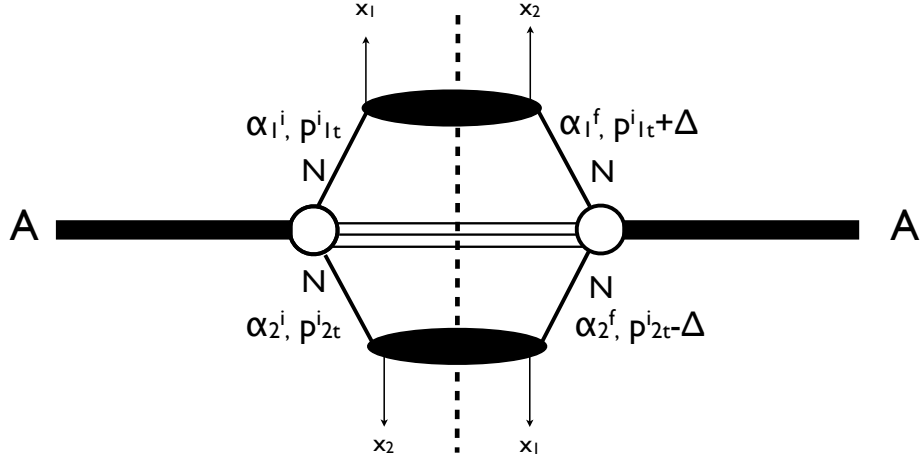


FIG. 3: Schematic representation of an interference contribution to the nucleus $_2$ GPD in which the two patrons of momentum fractions x_1, x_2 are drawn from two different nucleons in the amplitude, but in which the assignment between partons and active nucleons is swapped in the complex conjugate amplitude. As discussed in section V, these contributions (cases VII and VIII) are expected to be suppressed due to constraints on the longitudinal light-cone fractions of the two active nucleons.

and the normalized longitudinal parton correlation function

$$K(x'_1, x'_2) = \frac{G_p(x'_1, x'_2, 0)}{f_p(x'_1)f_p(x'_2)}. \quad (28)$$

The second term $SW(A)K(x'_1, x'_2)$ on the right hand side of (26) corresponds to the parametrically $A^{4/3}$ -enhanced contributions to the 4-jet double scattering cross section that we have discussed in section III. For a lead nucleus with Wood-Saxon density profile and $S = 15$ mb, one finds $SW(A) \sim 2.1$ [37]. This shows that these parametrically enhanced terms give indeed the largest contribution for sufficiently heavy nuclei. However, terms with linear A -dependence must not be neglected for numerical estimates, since they constitute about 1/3 of the four-jet cross section.

Equation (24) or (26) summarizes one of the main results of this paper. It demonstrates that any x'_1 - and x'_2 -dependence of the nuclear modification factor provides information about the longitudinal correlation (25) of two partons in the nucleon, since it constrains directly the ratio K . It is in this sense that the nucleus provides a non-trivial filter for analyzing the multi-parton structure of the proton. We further note that the effective area S is operationally defined as the ratio of the double hard 4-jet cross section and the product of two dijet cross sections in pp , and thus it can be a function of $S = S(x_1, x_2, x'_1, x'_2)$. However,

$$K(x'_1, x'_2) = \frac{R_{pA}^{4jet}(x_1, x_2, x'_1, x'_2) - 1}{SW(A)} \quad (29)$$

can depend only on the momentum fractions x'_1, x'_2 in the proton. Any deviation of the ratio (29) from unity would be an unambiguous signal of longitudinal momentum correlations in the proton. We note that the numerical analysis of [3], based on dominance of $3 \rightarrow 4$ processes, suggests $K \sim 1.2$ in the kinematic region under consideration. Vice versa, since the $3 \rightarrow 4$ contributions involve partonic $1 \rightarrow 2$ splittings in the nucleon and thus lead dynamically to longitudinal momentum correlations, any tight bound on $|K(x'_1, x'_2) - 1|$ will put significant constraints on the role of $3 \rightarrow 4$ processes. In the extreme case when $K(x'_1, x'_2) = 1$, one would have to conclude that the dynamical origin of the anomalously small effective area S is purely transverse. We argue that these considerations motivate an experimental study of the nuclear modification factor (24) and the corresponding longitudinal correlation function (29) in the upcoming pA run at the LHC.

We note as an aside that the corresponding result for proton-deuteron scattering can be calculated, based on the explicit form of the two-body deuteron form factor $F_D^{\text{double}}(\vec{\Delta}, -\vec{\Delta})$. Since the nucleons in the deuteron are strongly correlated in the center of mass frame, F_D^{double} is simply expressed through the one body deuteron form factor F_D as $F_D^{\text{double}}(\vec{\Delta}, -\vec{\Delta}) = F_D(4\Delta^2)$, similar to the case of the Glauber scattering, cf. [34]. Inserting this expression into (21) and performing the Δ -integral, one obtains $R_{pD}^{4jet} \simeq 1 + 1.07K$ for $x_i = O(0.01)$. We thank V. Guzey for performing this numerical integration and communicating the result. The case of pD scattering was considered recently in [30] in coordinate space formalism, though no numerical results were reported.

V. DOUBLE SCATTERING – INTERFERENCE TERM (CASES VII AND VIII)

Our discussion so far was based on a picture in which the nucleus is viewed as an independent superposition of nucleons, even if the parton distribution functions inside each nucleon may differ from those in a free nucleon. If one does not assume this picture, further interference contributions to the double hard four-jet cross section are conceivable. Here, we discuss the form of these contributions for completeness. We note at the beginning that we do not have a complete framework for calculating them, but we shall give some arguments of why we expect them to provide at best very small corrections to the expression (24).

As depicted in Fig. 3, it is possible to write down a diagram where in the initial state a parton with x_1 from the nucleon "1" is involved in the two to two process while this nucleon absorbs the parton with x_2 in the final state. This process changes the longitudinal light cone fractions $\alpha_{1/2}^{i/f}$ of the two active nucleons between the initial and final state,

$$\alpha_1^i - x_1 + x_2 = \alpha_1^f; \alpha_2^i + x_1 - x_2 = \alpha_2^f. \quad (30)$$

It thus implies that the momentum transfer $\vec{\Delta}$ now has also a nonzero longitudinal component, which in the nonrelativistic approximation can be written as

$$\vec{\Delta} = (\Delta_3 = (x_1 - x_2)m_N, \vec{\Delta}_t). \quad (31)$$

Consequently, in close analogy to the discussion of (14), the two-nucleon form factor takes now the form

$$F_A(\vec{\Delta}, -\vec{\Delta}) \approx F_A^2((x_1 - x_2)^2 m_N^2 + \vec{\Delta}_t^2) \approx \exp(-((x_1 - x_2)^2 m_N^2 + \vec{\Delta}_t^2) R_A^2/3) \quad (32)$$

in the mean field approximation. Here, the transverse factor is of the form of (14). The additional suppression factor $\exp(-(x_1 - x_2)^2 m_N^2 \cdot R_A^2/3)$ arises for significant differences in the longitudinal momentum fractions. For a typical value of the nuclear radius at large A , $R_A \sim 6$ fm, one finds a strong suppression in the range $|x_1 - x_2| \geq 0.03$. For $|x_1 - x_2| \leq 0.03$, this suppression factor is less important and it vanishes for $x_1 = x_2$. Naively, one may expect that in this phase space region such a contribution is thus enhanced. On general grounds, however, one may argue that this scattering diagram is also strongly suppressed in the range $|x_1 - x_2| \leq 0.03$, since a momentum exchange between nucleons should imply that the active nucleons in the nucleus are close in configuration space. Here, we do not attempt to estimate such potential further suppression effects further. We merely observe from (32) that such an additional contribution, if not negligible, will correct our main result (26),

$$R_{pA}^{4jet}(x_1, x_2, x'_1, x'_2) = 1 + S W(A) K(x'_1, x'_2) (1 + F_{\text{corr}}), \quad (33)$$

where the correction factor F_{corr} is expected to be small. In the case considered in (32), this correction factor will take the explicit form $F_{\text{corr}} = \text{const} \cdot \exp(-(x_1 - x_2)^2 m_N^2 R_A^2/3)$, where the proportionality factor is much smaller than unity.

VI. TWO NUCLEON – TO – ONE NUCLEON INTERFERENCE (CASE IX)

We finally consider the process where the two partons active in the double hard scattering are drawn from the same nucleon (with light-cone momentum fraction α_1) in the amplitude, but from two different nucleons (with light cone momentum fractions α_3, α_4) in the complex conjugate amplitude, see Fig.4. The momentum fraction of the spectator nucleon in the amplitude is denoted by α_2 . Longitudinal momentum conservation implies constraints on such processes. In particular, $\alpha_2 + \beta = \alpha_4$ and $x_2 \leq \beta$. Since the light cone momentum fractions α_i deviate from unity only by Fermi motion, this will constrain contributions of the diagram Fig.4 to relatively small momentum fractions x_2 . In addition, we expect that exchanges as depicted in Fig.4 can arise only between nucleons neighboring in impact parameter so that this diagram should be also suppressed by geometrical factors. Qualitatively, the limitations to sufficiently small x and a further suppression of geometrical origin are the same two suppression factors that we identified for the cases VII and VIII discussed above. To become more quantitative is difficult and lies outside the scope of the present work. We note, however, that diagram Fig.4 is closely related to the leading twist shadowing correction, discussed in Ref. [38]. We now turn to a more general discussion of the corrections to (24) that may arise from the nuclear dependence of parton distribution functions.

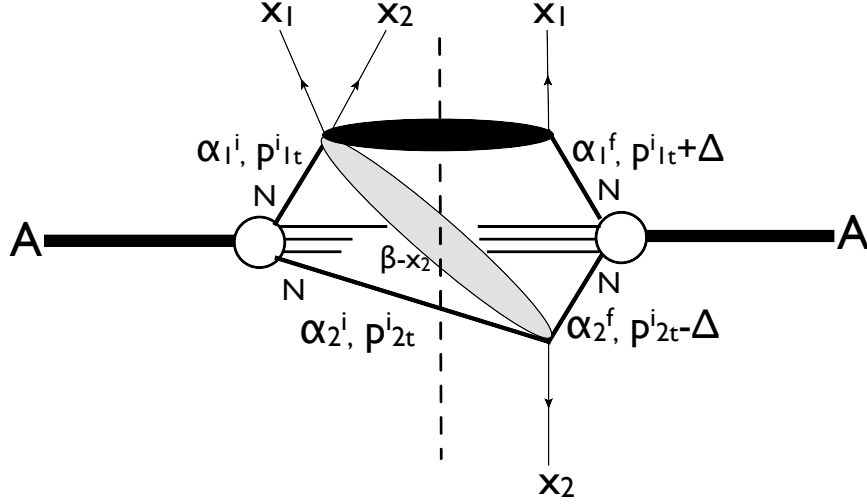


FIG. 4: Schematic representation of an interference contribution to the nucleus $_2\text{GPD}$ in which the two partons are drawn from a single nucleon in the amplitude while the parton '2' is associated to a different nucleon in the complex conjugate amplitude. This process requires the exchange of additional constituents between both active nucleons, as indicated by the grey blob.

VII. NUCLEAR DEPENDENCE OF SINGLE PARTON DISTRIBUTION FUNCTIONS

In our discussion so far, we have neglected nuclear modifications of the single parton distribution functions. Standard parametrizations of these modifications are based on a linear relation between the parton distributions $f_{i/p}(x, Q^2)$ in a proton and the parton distributions $f_{i/A}(x, Q^2)$ per nucleon in a nucleus [38, 40–43]

$$f_{i/A}(x, Q^2) = R_i^A(x, Q^2) f_{i/p}(x, Q^2). \quad (34)$$

In all parametrizations that are consistent with linear Q^2 -evolution [38, 40–43], the npdf factors $R_i^A(x, Q^2)$ at fixed x approach unity with increasing Q^2 . Since the jet production considered here is a hard process with typically $Q^2 \gg 100 \text{ GeV}^2$, we expect $|R_i^A(x, Q^2) - 1| < 3\%$ over the entire x , and Q^2 -range relevant for four-jet production at the LHC. This assumption can be verified experimentally by checking that the nuclear modification factor for dijet production,

$$R_{pA}^{2\text{jet}} \equiv \frac{d\sigma_{2\text{jet}}^{pA}}{dt_1} / A \frac{d\sigma_{2\text{jet}}^{pp}}{dt_2}, \quad (35)$$

deviates from unity by less than 3% in the kinematical range used to measure the four-jet cross sections. An $\mathcal{O}(3\%)$ uncertainty from “npdf corrections” to the dijet cross section entering the norm in (24) is expected to result in an $\mathcal{O}(6\%)$ uncertainty on the nuclear modification factor for double hard four-jet production. The central question is whether a correction to (24) of this order is small enough to be neglected in the analysis of the longitudinal two-parton correlation function $K(x_1, x_2)$. This depends on how the size of the deviation of $K(x_1, x_2)$ from unity compares to the size of the npdf correction of (24), and we shall distinguish below two different cases.

Let us discuss, however, first how the nuclear modification of longitudinal parton momenta in the nucleus can be taken into account in the formulation of $_2\text{GPDs}$. To this end, one needs to specify correction factors for the three contributions to the nuclear $_2\text{GPD}$ listed on the right hand side of equation (4). In principle, this requires more information than what is contained in the npdf-fits based on (34). Adopting the picture of the nucleus as an incoherent superposition of nucleons that have parton distributions shifted according to (34), one arrives at corrections of the simple form

$$G_A^{2N}(x_1, x_2, 0) \longrightarrow R_i^A(x_1, Q^2) R_j^A(x_2, Q^2) G_A^{2N}(x_1, x_2, 0), \quad (36)$$

$$G_A^{\text{double}, 1N}(x_1, x_2, 0) \longrightarrow R_i^A(x_1, Q^2) R_j^A(x_2, Q^2) G_A^{\text{double}, 1N}(x_1, x_2, 0). \quad (37)$$

In principle, nuclear pdfs depend also on the impact parameter b (see Ref. [38, 39] for first parametrizations), and this b -dependence could be accommodated in (36) and (37). This, however, will be a small correction on top of the correction discussed here, and we neglect it in the following.

The third contribution $G_A^{single,1N}$ to the nuclear $_2\text{GPD}$ does not have an npdf correction factor of this form. Rather, since the arguments x_1, x_2 in $G_A^{single,1N}$ result dynamically from the splitting of a single parton, the npdf corrections will be determined by an integration over the available phase space that may be written formally as

$$G_A^{single,1N}(x_1, x_2, 0) \longrightarrow O(R_i^A(x_1 + x_2, Q^2)) G_A^{single,1N}(x_1, x_2, 0). \quad (38)$$

To determine npdf corrections to (24) in the most general case, one would have to specify (38) fully and then repeat the calculations in sections II-IV based on equations (36)-(38). Because of the more complicated form of (38), the explicit results for the npdf corrections to (24) would be relatively involved. Here, we restrict our discussion by considering two limiting cases:

First, we observe that if (38) makes a numerically important contribution to the total nuclear $_2\text{GPD}$ (4), then longitudinal two-parton correlations in the nucleus are expected to be large. This is so, since (38) is dynamically generated by a perturbative parton branching that inevitably results in significant longitudinal correlations. Indeed, according to the model of [3] one can expect a $\sim 20\%$ deviation of $K(x_1, x_2)$ from unity if the double hard four jet cross section is dominated by $3 \rightarrow 4$ processes. Therefore, if the term (38) is relevant, we expect that $K(x_1, x_2) - 1 > 0.1$ and that the npdf-corrections to (24) are small. In this case, $K(x_1, x_2) - 1$ is much larger than the expected npdf corrections and it can thus be extracted safely from (29) without taking npdf corrections into account. We emphasize that the validity of this procedure can be checked experimentally by measuring the nuclear modification factor for dijet production in pA.

Alternatively, if an experimental determination of $K(x_1, x_2) - 1$ from (29) yields values $K(x_1, x_2) - 1 \ll 0.1$, the contribution of the $1 \rightarrow 2$ splitting term (38) to the nuclear $_2\text{GPD}$ can be expected to be small, and one can justify hence the use of a simplified npdf correction of the form $_2G_A(x_1, x_2) \rightarrow R_i^A(x_1, Q^2) R_j^A(x_2, Q^2) {}_2G_A(x_1, x_2)$. In this case, the npdf corrections to the contributions (7), (19) and (21) amount to multiplying all three expressions with two powers of the nuclear modification factor for dijets, R_{pA}^{2jet} . As a consequence, the right hand side of (24) gets multiplied by two powers of R_{pA}^{2jet} , and the npdf correction to (29) takes the form

$$K(x'_1, x'_2)|_{\text{npdf corrected}} = \frac{R_{pA}^{4jet}(x_1, x_2, x'_1, x'_2)/(R_{pA}^{2jet})^2 - 1}{SW(A)}. \quad (39)$$

Experimentally, comparing the values obtained from (29) and (39) provides a direct way of estimating the importance of npdf corrections on the interpretation of K as a normalized longitudinal two-parton correlation function.

VIII. CONCLUSIONS.

In summary, by analyzing $_2\text{GPDs}$ for the nucleus in the many nucleon approximation, we have derived a compact expression (24) for the nuclear modification factor of the double-hard four jet cross section in pA collisions. Based on this main result, we have outlined an experimental strategy for determining the normalized longitudinal two-parton correlation function $K(x_1, x_2)$ in the proton by combining data from pp and pA collisions. Finally, we have discussed how nuclear modifications of parton distribution functions can be taken into account in this analysis.

In early 2013, the LHC is scheduled for a 4-week-long proton-nucleus run. The main motivation for this pA program at the LHC is to constrain the parton distributions in the nucleus and to provide important benchmark measurements for the LHC heavy ion programme. As illustrated by the calculation of $K(x'_1, x'_2)$ in this paper, however, proton-nucleus collisions may also contribute to further constrain the multi-parton structure of the proton, thus probing the proton in nuclear collisions rather than probing the nucleus in collisions with protons.

Acknowledgements

We thank Leonid Frankfurt, Vadim Guzey and Daniele Treliani for useful discussions. The research of MS was partially supported by grant from the US Department of Energy.

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